Day 1 Task Descriptions

1. Linear control of a coupled cavity

For the LIGO interferometers to be sensitive to gravitational waves, separations between mirrors (i.e. the lengths of the optical cavities they form, a.k.a. Degrees of Freedom or DoFs) have to be maintained within a very precise, narrow range. In practise, this means that we have to use *feedback control* to sense the positions of the mirrors and then actuate (move) them to maintain the required resonant conditions.

LIGO uses a technique known as frontal phase modulation to sense the various DoFs. You have already encountered a simple example of this in the Pound-Drever-Hall (PDH) locking of a *single* optical cavity task at the workshop. The situation in LIGO is more complex because of the need to sense the lengths of *multiple, coupled* optical cavities. In this task, you will have to determine a length for the power recycling cavity (PRC) that will allow the lengths of the coupled PRC and arm cavity to be sensed using the PDH scheme with a *single* RF phase modulation applied to the carrier field.

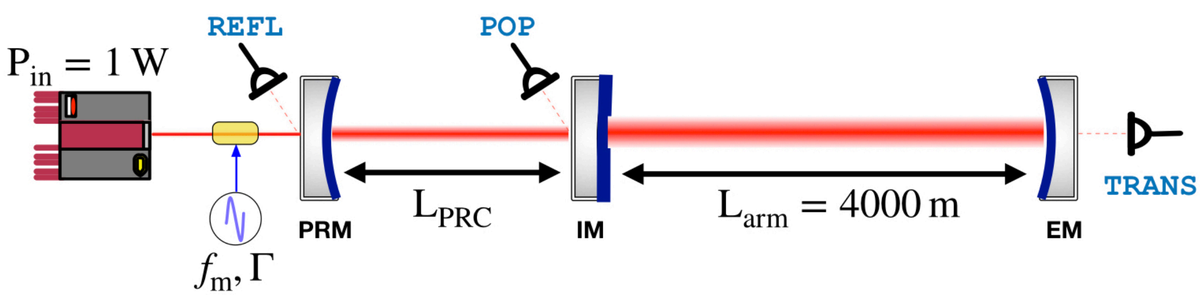

Fig 1: Schematic representation of a coupled cavity system

As you have seen in the workshop, the amplitude reflectivity for a 2-mirror Fabry-Pérot cavity is

$$r(\varphi) = \frac{-r_i + r_e e^{-2i\varphi}}{1 - r_i r_e e^{-2i\varphi}}$$

Where $\varphi = \frac{\omega l}{c}$ is the phase a field accrues going one-way along the cavity. If the complex argument $arg(r_i r_e e^{-2i\varphi}) = 2n\pi,\ n \in Z$, the cavity is said to be *resonant.* If $arg(r_i r_e e^{-2i\varphi}) = (2n+1)\pi$, the cavity is said to be *anti-resonant*. These continue to hold if $r_e = |r_e|e^{i\theta}$ is a complex number, as is the case for optical cavities in general.

1. The length of the arm cavity is chosen to be 4km to optimize the sensitivity of the detector to gravitational waves in a certain frequency band (keeping other practical constraints like cost, land availability etc in mind). For this choice of arm cavity length, the task is to determine the length of the PRC that will allow us to use a 9 MHz phase modulation sideband for control.
    a. From Figure 1, we can model the coupled cavity as a two mirror cavity, whose end mirror (called a compound mirror) is the 4 km long arm cavity. The PRC macroscopic length should be set such that f_m and the carrier field are *both* resonant in the PRC when the arm cavity is resonant for the carrier field. Rearranging the equations given above, you should find that candidate PRC lengths are given by $L_{PRC} = (n + \frac{\theta}{2\pi})\frac{c}{2f_m}$, where $f_m = 9.100230$ MHz is the modulation frequency and θ is the complex argument of the arm cavity reflectivity at this frequency. To be consistent with other design considerations, find a candidate PRC length that lies between the range 45m -70m.
    b. Set up a *plane waves* Finesse model of a LIGO-like power-recycled arm cavity (see the sketch above), with L_{PRC} being what you determined. Other relevant parameters are in the following table:

Input Mirror (IM) power transmissivity	T_I	0.014
End Mirror (EM) power transmissivity	T_E	5e-6
Power-recycling mirror (PRM) power transmissivity	T_P	0.03
Arm cavity	Length [m]	4000
Modulation frequency	f_m [MHz]	9.100230
Modulation depth	Γ [rad]	0.1

    c. With the arm cavity held resonant for the carrier, plot the power (at the carrier and sideband frequencies) circulating in each cavity as a function of the microscopic tuning of the PRM. What tuning should the PRM have so that it has the desired effect? Keep in mind that Finesse uses an i-on-transmission convention for mirror transfer matrices.
    d. Confirm *numerically* that for the tuning of the arm cavity and PRM determined above, the macroscopic length you have chosen results in the maximum simultaneous power build up in the PRC for all the fields considered above.

2. If you have not already done so, add photodiodes to your Finesse model at the REFL, POP, and TRANS (see sketch above) that can be used to measure the PDH error signals at these ports. For the POP port, in an actual optical setup, we will only be able to access a few parts per million of the circulating PRC field without degrading the buildup, (there would be no optical cavity if we put a photodiode inside that blocks the beam). Some options are to use the wedged AR coated surface of the IM, or the leakage through a highly reflective folding mirror (in which case the PRC would be a *folded* cavity). However, for our simulation, we can avoid these practical complications by setting up a system as in the sketch above, with a photodiode “inside” the PRC. However, you should scale these signals by 1/20,000 to mimic the size of the signals we would get in an actual optical setup.
3. Plot the PDH error signal for the case that the arm cavity is held resonant for the carrier, as a function of the PRM microscopic tuning. Sweep through at least one full linewidth of the carrier resonance. Be sure to optimize the demodulation phase to maximize the sensitivity.
4. Plot the frequency dependence of the PDH error signals.
    a. What is the shot noise sensing limit of measuring displacement noise of the *PRM* and *EM* using the PDH scheme with the parameters above? You should investigate PRM motion and ETM motion separately, but be sure to comment on the shape and relative magnitudes of the signals at different ports.
    b. How does this change as a function of the modulation depth? Some representative values you may try are $\Gamma = 0.01$ rad, $\Gamma = 0.1$ rad and $\Gamma = 0.3$ rad. What are some possible reasons to confine ourselves to the small modulation depth regime?

References

You may consult [this document]() on the Hackathon resource page for a more general overview of how the recycling cavity lengths are chosen for an optical topology closer to that of a LIGO interferometer.

2. A mode-matched coupled cavity

We will now introduce higher-order modes (HOMs) into the mix, meaning there are a few extra considerations to take into account when choosing optical parameters. These include:

- **Mode matching** — As you have seen from the mentoring notebook on Gaussian beams, an optical cavity has an associated *eigenmode*. This is the waist size and position, or alternatively q, of a beam that will resonate within the cavity. If a beam with q different to that of the cavity eigenmode enters the cavity, some scattering into higher-order modes will occur—this is called a *mismatch*.

- **Stability** — You have also seen the parameter g, which is related to the cavity's stability m by $g = \frac{m+1}{2}$. This must satisfy $0 \le g \le 1$ (or $-1 \le m \le 1$) for the cavity to be stable.

In this task, you will build a model similar to that in the previous task, and determine the PRC parameters required to produce a well-matched, stable coupled cavity setup.

1. Make a Gaussian model of a LIGO-like arm cavity using the parameters listed below:

Input Mirror (IM)	T_I	0.014
	RoC [m]	1940
End Mirror (EM)	T_E	5e-6
	RoC [m]	2245
Arm cavity	Length [m]	4000

2. Investigate the effects of cavity stability on the arm:
   a. Using the `cp` detector or otherwise, calculate the g-factor of the arm cavity.
   b. Introduce a small mismatch into your cavity, by tilting EM by .1 μrad in x, and plot the circulating power as a function of EM tuning.
   c. Move the cavity close to instability, by setting the RoC of IM to 1760m. What is the value of g now?
   d. Plot the circulating power vs. EM tuning again. What happens? Why is this undesirable?
3. Use a beam parameter detector to calculate the curvature of the beam at the PRM location you found in Task 1 (Linear control of a coupled cavity). Be sure to reset the RoC of IM to 1940m and remove any static misalignments.
4. Add the power-recycling mirror (PRM) before the IM, with $T_P = 0.03$. Mode-match the power recycling cavity (PRC) to the arm using your answer from above as a starting point, adjusting the PRM's curvature to bring the mismatch below 0.1%.

N.B. In order to determine when the cavities are well mode-matched, Finesse & PyKat provide a couple of ways to measure the mismatch:

- The `mismatches` command in Finesse can calculate and print the mismatches at each component. It has the syntax `mismatches [limit] [n]`, where `limit` is the minimum mismatch value to show, and `n` is a number controlling the behaviour of the command. You can see a more complete description by running `kat -hh` from a terminal.
    - By default, only the mismatches for the first data point on your `xaxis` are shown. You should therefore be careful to ensure that `noxaxis` is set, or alternatively use `mismatches [limit] 8` to show the mismatches at every step of the calculation.
    - Similarly to the `trace` command, you must call `print(out.stdout)` on the output of running the simulation to see the mismatches.
- PyKat provides a command `pykat.ifo.mismatch_cavities(kat, node)`, which will return a tuple of `mmx, mmy, [list of cavity eigenmodes]`, where `mmx` & `mmy` are the mismatches in x & y respectively.
    - This will ignore any `xaxis` commands in your file by temporarily setting `kat.noxaxis = True`

5. Plot the power circulating in each cavity, for the same tunings as Task 1 (Linear control of a coupled cavity). What is the gain of the PRC (power-recycling gain, or PRG) at this tuning for (i) the carrier field, and (ii) the sideband field?
6. Calculate the g-factor of the PRC. Is it possible to adjust the RoC of the PRM such that g_{PRC} ~ 0.8, without compromising the PRG, arm power or mode-mismatch values?
7. Add in the modulators you used in Task 1 (Linear control of a coupled cavity) and confirm that the PDH error signals derived there still behave as expected (i.e. confirm that the optical gain and frequency dependence of the PDH error signals in response to PRM and EM motion are as they were).