Appendix B

Modelling Complex Interferometers in Finesse

In this appendix I provide an overview of how Finesse can be used to model the core optics of a Dual-Recycled Michelson Interferometer with Fabry-Perot arm cavities (DRFPMi). The file is tuned and mode-matched as it is built, beginning with a single arm cavity and finishing with the quantum-limited sensitivity curve of the interferometer using DC readout. It is assumed that key parameter values, such as cavity lengths and mirror curvatures, have been pre-determined here, however one may explore each of these parameters as they are integrated into the model. In this case, it can be useful to first build a plane-wave model of the detector, then include Gaussian beams and associated properties later.

I first provide an overview of the procedure, then demonstrate the process using the example of an ET-120K design that has 6690 m radii of curvature on all test masses (see chapter **??** for details of this design).

B.1 Overview

In order to explore the behaviour of current detectors and test out proposed future detector designs, a base Finesse file (with extension `.kat`) is written, containing all of the main optics in their optimal design configuration. For example, there are currently 3 main `kat` files for LIGO: one matching the Advanced LIGO design study, and one for each of the Livingston and Hanford sites.

While the first shows how the LIGO detectors were intended to work in the original design, the site files are updated to match parameters at the sites as closely as possible. The aLIGO file is therefore useful for understanding general behaviours in a more ideal interferometer case, while the site files are useful diagnostic tools.

We build the interferometer model in stages so that the behaviour of each element can be understood and checked along the way:

0. Before beginning a complex FINESSE model, it is useful to plan the file: collect together all needed constants, draw a sketch of the layout such as shown in figure B.1, and name the key components, lengths and nodes in a way that is memorable and adaptable.
1. **Model a single arm of the interferometer** including any optics in the path from the beamsplitter (BS) to the cavity. By default the laser wavelength is set to 1064 nm; this can be overridden using the `lambda` command. If the laser beam shape is not the fundamental Gaussian mode (or plane wave), this can be set using `tem` commands. Define the arm cavity using the `cav` command. By default the model is plane-wave; you can specify the order of higher order modes (HOMs) to include in the model using the `maxtem` command. At any time, the model can be swapped back to the plane waves case by setting `maxtem off`. If using Gaussian beams, check the cavity stability and other values of interest such as the spot sizes on the test masses by using `cp` commands or `trace 2`. Additional properties of the test masses, such as their mass, suspension transfer function, and radius (to include clipping effects) should be added as attributes using the `attr` command once the core optical model is built, so that their effects do not get confused with the base optical response.
2. **Introduce the Michelson and second arm cavity**. In an ideal case the two arms are identical copies of each other, so here we introduce the beamsplitter and link the two together. Now the behaviour of the Michelson with Fabry-Perot arms (FPMi) can be explored. The *CARM* and *DARM* degrees of freedom can be introduced as functions of the input mirror tunings, then used to plot the power at the exit port and in the arms as the common and differential displacements of the test masses are scanned. We can now also set the Michelson at the operating point by checking the beamsplitter tuning is such that for zero displacement, no light is detected at the exit port of the interferometer. If the beamsplitter is realistic (i.e. thick – modelled as a set of three beamsplitters connected by spaces with the optical path length and refractive index of the beamsplitter material, see figure B.2), the spaces from the BS to the arm cavity input mirrors may need to be adjusted to ensure that the the power

circulating in the two arms is matched. This also improves the mode matching in the FPMi.

3. **Add the power recycling mirror** (PRM), and define the additional cavities this introduces with the X and Y-arms. This is the Power-Recycled FPMi (PRFPMi). The interferometer should now reach (or exceed, given that this is a 'perfect' model) the design arm circulating power. The effect can be observed by plotting powers as *CARM* and *DARM* are swept, as before, or by directly outputting the circulating power on resonance. If Gaussian beams are used, the power recycling cavity (PRC) can be mode matched to the FPMi by matching its curvature to the curvature of the field reflected from FPMi at the target position of the PRM. Further optimisation can be achieved by adjusting the curvature of the PRM or length of the space from the PRM to BS such that the arm power is maximised.

4. **Add the signal recycling mirror** (SRM) in the same way, to make the Dual Recycled FPMi (DRFPMi). In order to see the effect of signal recycling, it is more useful to look at the quantum-limited sensitivity curves. The masses of the input and end mirrors, and that they are suspended, must now be taken into account so that the radiation pressure curve can be calculated. A differential frequency signal, mimicking a gravitational wave, is then applied to the two arm lengths and the quantum-limited sensitivity calculated as this frequency is scanned. At this stage, we assume DC readout will be used, and therefore apply a small offset to the ETM tunings (the 'DC offset') so that we have a detectable signal. Broadband sensitivity is achieved using RSE, when the signal recycling cavity (SRC) is tuned to be anti-resonant with the arms. Usually this occurs for an SRC tuning of 90°. In cases where higher order optical modes are included in the model, the DC offset should be set such that the mode containing the gravitational wave signal is the dominant power source at the dark port; if other modes contain more power, these will dilute the signal and decrease the detector SNR.

The model now contains the 'core optics' of a DRFPMi gravitational wave detector. In reality, there will be many additional parts and features. These can be added to the model in a similar stepwise fashion when they become important to the behaviour of the detector. Potential additions include:

- Sensors, including photodiodes (`pd`) to measure the power at various locations, and demodulated (`pd[n]`) and partitioned photodiodes (via `pdtype`) like quadrant or bullseye sensors to measure error signals;

- Modulators (`mod`), usually added in the input path between the laser and PRM, so that error signals for the linear, angular and mode matching degrees of freedom can be modelled and control schemes designed;
- Schnupp Asymmetry – deliberately including a macroscopic length difference between the BS-ITMX and BS-ITMY path lengths so that modulated optical fields are partially transmitted to the output port of the detector;
- Input Mode Cleaner(s) (IMCs), included before the PRM to ensure that only the desired beam shape enters the detector[1];
- Output Mode Cleaner(s) (OMCs) to separate the optical mode containing the gravitational wave signal from HOMs generated in the detector;
- Isolators (`isol`), for injection of squeezed light and to prevent unwanted back-reflections;
- Squeezers and filter cavities to enable frequency-dependent squeezing;
- Homodyne detection – removing the DC offset and instead beating the field at the output port with a local oscillator at the same frequency as the main laser to extract the gravitational wave signal;
- Other new optical technologies for testing.

[1] In Finesse the beam is 'clean' by default, however in reality the beam from the laser will include some HOM content that should be removed before entering the detector. Including the IMC in the detector model means that effects such as mode mismatching between the IMC and core detector can be explored